**PERFORMANCE ANALYSIS OF TWO MODIFIED BROYDEN METHODS FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS**

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**Abstract*:*** *In this paper an analysis on the performance of two modified Broyden method is presented: Broyden – like Method (BLM) and Trapezoidal Broyden Method (TBM) . Four test problems with standard initial points were used to compare the performance of the two methods in terms of CPU time and Number of Iterations. Numerical results have shown that there is little difference between the two methods in terms of Number of Iterations. Further analysis using performance indices has also shown that TBM is superior to BLM in terms of CPU time.*

# Keywords: Trapezoidal Broyden Method, Broyden-Llike Method, Broyden Method, Performance analysis.

1. **Introduction**

Many tasks in computational mathematics involve the numerical solution of systems of nonlinear equations. Broyden’s method which is a modification of the famous Newton method is commonly used. In this method instead of evaluating the Jacobian matrix at each iteration an approximate Jacobian  is used, and is updated with another matrix  by adding a rank-one matrix to an existing factorized approximation [[1](#_ENREF_5)]. The convergence rate of the Broyden’s method is super linear and the floating point operation is against  for Newton’s method [[2](#_ENREF_6)]. Similarly, the Broyden’s method satisfies the secant equation.

In this paper an analysis on the performance of two modified Broyden method is presented: Broyden – like Method (BLM) and Trapezoidal Broyden Method (TBM) as proposed by [1, 2]. The two methods improved the classical Broyden by reducing the number of iteration it performs before it converges. The rest of the paper is arranged as follows. In section 2 the two methods are presented, numerical results are discussed in section 3.The conclusion is discussed in section 4.

2. **Algorithms of the two Broyden like methods**

The first method Broyden Like Method (BLM), use Jacobian matrix for first iteration instead of identity matrix (as it is done in classical Broyden method) and continue with classical Broyden method in the subsequent iterations using the Broyden updating formula [1]. The idea is to carry more information about the function being evaluated so as to fasten the convergence. The Algorithm is as follows

**Algorithm 2.1** ( Broyden Like Method (BLM)

**Step 1**: Choose an initial guess , let

**Step 2**: Compute if stop else Go to step 3

**Step 3**: Compute the initial Jacobian matrix

Compute via Newton’s method

**Step 3**: Compute

**Step 4**: Compute ,and

**Step 5**: Update the current Broyden’s matrix for

**Step 6**: Compute the next point via Broyden method.

**Step 7**: Repeat steps 3 to 7 and continue with the next k until is satisfied.

The second method Trapezoidal Broyden Method (TBM) employs a two point predictor - corrector approach where the classical Broyden method is the predictor and the new method (TBM) is the corrector. The algorithm is as follows:

**Algorithm 2.2** (Trapezoidal Broyden Method)

**Step 1**: Given initial guess, let and

**Step 2**: Compute if is satisfied stop.

**Step 3**: Compute were .

**Step 4**: Compute using

Where,

**Step 4**: Compute using

,

**Step 5**: Set and go to Step 2.

For details of the two methods see [1,2].

**3. Numerical Result**

In this section the result of the computational comparison between the two methods based on the number of iteration and CPU time is presented. Four test problems with standard initial points are used, and the dimension is between 15 to 1000.The test problems used are:

Problem 1 Spares function of Byeong [3]

Problem 2 Extended Spares system of Byeong [3]

Problem 3 Trigonometric system of Byeong [3]

Problem 4 Extended Trigonometric Exponential system [4]

The results are computed using Matlab 7.1 on a double precision computer. The stopping criteria used is Table 3.1 shows the numerical result base on number of iterations and CPU time.

**Table 3.1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| PROBLEM | n | TRAPEZOIDAL BROYDEN METHOD(TBM) | | BROYDEN - LIKE METHOD  (BLM) | |
| NI | CPU Time | NI | CPU Time |
| 1 | | 15  50  100  200  500  1000 | 4  4  4  4  4  4 | 0.001950  0.040103  0.073283  0.084622  0.223840  0.866357 | 4  4  4  4  4  4 | 0.004123  0.010032  0.063645  0.068661  0.750773  5.352363 |
| 2 | | 15  50  100  200  500  1000 | 3  4  4  4  4  4 | 0.002619  0.021108  0.026877  0.062317  0.284324  0.877542 | 4  4  4  4  4  4 | 0.003124  0.025802  0.060135  0.082321  0.738822  5.388611 |
| 3 | | 15  50  100  200  500  1000 | 9  10  10  10  11  11 | 0.006759  0.016374  0.045415  0.173818  0.688665  3.245712 | 11  11  12  12  12  13 | 0.003325  0.034023  0.063816  0.135442  0.978636  6.417418 |
| 4 | | 15  50  100  200  500  1000 | 4  4  5  5  5  5 | 0.001813  0.011907  0.042648  0.047769  0.206503  0.864835 | 4  4  4  4  4  4 | 0.013131  0.060536  0.198443  0.288024  1.654416  9.768337 |

Table 3.1 shows that there is little difference between the two methods in terms of number of iterations. A further analysis was carried out base on comparison indices of [5] that is the robustness, efficiency and combined efficiency indices of the two methods base on CPU time, and is reported as shown in the following graphs.

***Figure 3.1*** *Efficiency profile of HBM, and TBM methods as the dimensions increase*

*(in terms of CPU time)*

***Figure 3.2*** *Robustness profile of HBM, and TBM methods as the dimensions increase (in terms of CPU time)*

***Figure 3.3*** *Combined Robustness and Efficiency profile of HBM, and TBM methods*

*as the dimensions increase (in terms of CPU time)*

Graphs 3.1 – 3.3 shows that TBM is superior to BLM in terms of CPU time especially as the dimension increases. This can be attributed to the fact that BLM requires the computation of the Jacobian matrix at the first iteration which becomes more computationally costly as the dimension increase.

Therefore we can conclude that TBM is a better solver especially when handling large scale systems.

**4. Conclusion**

A comparison of the performance of two modified Broyden methods (TBM and BLM) was carried out in this paper. Computational results as presented in Table 3.1 and Fig 3.1 – 3.3 have shown that TBM is superior to BLM in terms of CPU time. From the result it is concluded that TBM is the best solver of system of nonlinear equation as compared to BLM especially large scale.

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